Closing *Wed*: HW_7A,7B (7.5, 7.7, 7.8)

Note: Exam 2 is **Thursday**!!!

Covers 6.4, 6.5, 7.1-7.5, 7.7, 7.8

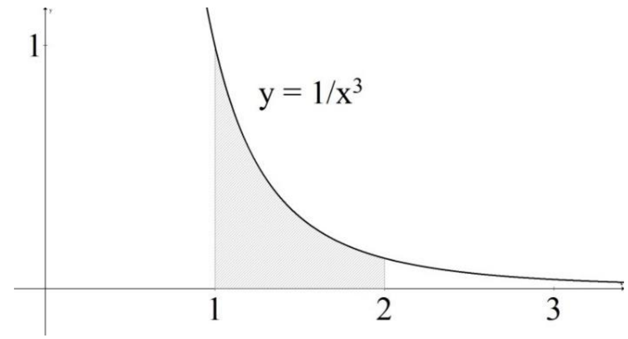
The exam will roughly look like this:

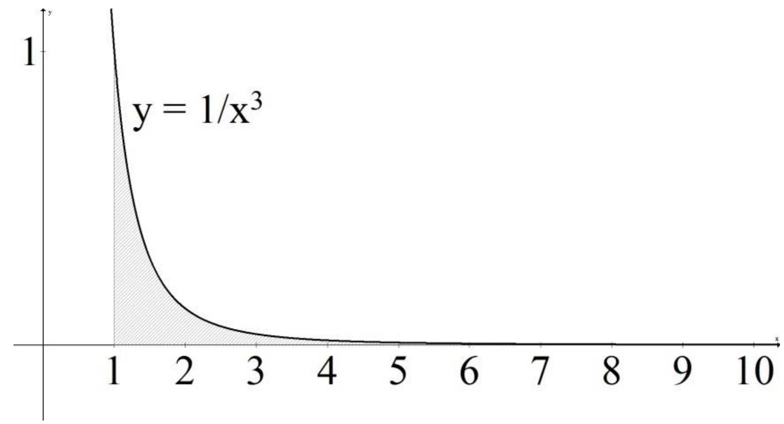
First 3 pages: 6 integrals (*ALL* types)

4th page: 6.5, 7.7 and/or 7.8

5th page: 6.4

(8.1 Arc Length is NOT on our midterm)





7.8 Improper Integrals

Motivation: Consider the function $f(x) = \frac{1}{x^3}$. Compute the area under the function from…

1. x = 1 to x = t
2. x = 1 to x = 10
3. x = 1 to x = 100

Def'n: *Improper type 1 - infinite integral of integration*

$$\int_a^{\infty} f(x)dx = \lim_{t\to\infty} \int_a^{t} f(x)dx$$

$$\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty} \int_t^{b} f(x)dx$$

If the limit exists and is finite, then we say the integral *converges*.

Otherwise, we say it *diverges*.

Example:

$$\int_1^{\infty} \frac{1}{x^3}\,dx =$$

Example:

$$\int_{-1}^{\infty} e^{-2x}\,dx =$$

Example:

$$\int_{1}^{\infty} \frac{1}{x}\,dx =$$

Def'n:

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{r\to-\infty} \int_{r}^{0} f(x)dx + \lim_{t\to\infty} \int_{0}^{t} f(x)dx$$

In this case, we say it *converges* only if both limits separately exist and are finite.

Example:

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

Def'n: *Improper type 2 - infinite discontinuity*

If $f(x)$ has a discontinuity at x = a, then

$$\int_a^b f(x)dx = \lim_{t \to a^+} \int_t^b f(x)dx$$

If $f(x)$ has a discontinuity at x = b, then

$$\int_a^b f(x)dx = \lim_{t \to b^-} \int_a^t f(x)dx$$

If the limit exists and is finite, then we say the integral *converges*.

Otherwise, we say it *diverges*.

Example:

$$\int_0^1 \frac{1}{\sqrt{x}} dx =$$

Example:

$$\int_0^2 \frac{x}{x-2}\,dx =$$

If $f(x)$ has a discontinuity at $x = c$ which is **between** a and b, then

$$\int_a^b f(x)dx = \lim_{r \to c^-} \int_a^r f(x)dx + \lim_{t \to c^+} \int_t^b f(x)dx$$

In this case, we say it *converges* only if both limits separately exist and are finite.

Example:

$$\int_0^{\pi} \frac{1}{\cos^2(x)} dx =$$

Limits Refresher

1. If stuck, plug in values "near" t.

2. Know your basic functions/values:

$$\lim_{t\to\infty} \frac{1}{t} = 0,$$

$$\lim_{t\to\infty} \frac{1}{e^t} = 0\,,$$

$$\lim_{t\to\infty} \ln(t) = \infty.$$

3. For indeterminant forms, use algebra and/or L'Hopital's rule

Examples:

$$\lim_{t\to 1} \frac{t^2+2t-3}{t-1} =$$

$$\lim_{t\to\infty} \frac{\ln(t)}{t} \quad =$$

$$\lim_{t\to\infty} t^2 e^{-3t} =$$

Example:

$$\int_0^{\infty} xe^{-x^2}dx$$

Example:

$$\int_{0}^{\infty} xe^{-2x}dx$$

Aside:
A few general notes on **comparison**:

Suppose you have two functions $f(x)$ and $g(x)$ such that

$$0 \le g(x) \le f(x)$$

for all values of x.

(a) If $\int_a^\infty f(x)dx$ converges,
then $\int_a^\infty g(x)dx$ converges.

(b) If $\int_a^\infty g(x)dx$ diverges,
then $\int_a^\infty f(x)dx$ diverges.

You can verify that

$$\int_1^\infty \frac{1}{x^p}\,dx\,, \qquad \text{converges for } p > 1.$$

$$\int_1^\infty e^{px}\,dx\,, \qquad \text{converges for } p < 0.$$

You can compare off of these to sometimes quickly tell if an integral converges/diverges (without computing)

Example:

$$\int_1^\infty \frac{1}{x^4 + x}\,dx \text{ converges}$$

because

1. $\frac{1}{x^4+x} < \frac{1}{x^4}$ for all $x > 1$, and
2. $\int_1^\infty \frac{1}{x^4}\,dx$ converges.

Example:

$$\int_1^\infty \frac{2 + \cos(x)}{x}\,dx \text{ diverges}$$

because

1. $\frac{2+\cos(x)}{x} > \frac{1}{x}$ for all $x > 1$, and
2. $\int_1^\infty \frac{1}{x}\,dx$ diverges.